Counting nodes in chess

Richard Bean

Institute for Studies in Theoretical Physics and Mathematics
PO Box 19395-5746
Tehran, I.R. Iran

Abstract

The number of nodes in the game of chess from the starting position has previously been calculated up to a depth of 9 ply. Here, this calculation is extended to 10 ply, and the number of unique nodes (taking into account *en passant* and castling status) up to 8 ply is also given. Previously, this value was known up to 6 ply, though the value given for the sixth ply was in error.

The following table gives the number of nodes at a depth of up to 10 ply in chess. One ply is a move by either white or black. This value can be calculated using free chess software such as Crafty [4].

The 10 ply calculation here took approximately one week on a 1.9Ghz Pentium 4; the main speed optimization was to calculate and store the unique nodes at each ply up to 8 ply before calculating the number of nodes at 10 ply. Other programs increase the speed of the process by only generating legal moves, and thus making and unmaking the moves at the last ply becomes unnecessary.

We denote the number of ply by n, the number of nodes at depth n by $N(n)$ and the number of unique nodes at depth n by $U(n)$.

n	$N(n)$	$U(n)$	$N(n)^{1/n}$	$U(n)^{1/n}$
0	1	1	-	-
1	20	20	20.000	20.000
2	400	400	20.000	20.000
3	8902	5362	20.725	17.503
4	197281	72078	21.075	16.385
5	4865609	822518	21.748	15.242
6	119060324	9417681	22.180	14.532
7	3195901860	96400068	22.793	13.822
8	84998978956	988187354	23.236	13.315
9	2439530234167	-	23.789	-
10	69352859712417	-	24.216	-

The values of N(5) to N(8) were given by Steven J Edwards on the Usenet newsgroups rec.games.chess.* [1] and the Computer Chess Club [2] message

board from 1993 to 1998. N(9) was calculated by Michel Langeveld in June 1999 and the result was posted on the Computer Chess Club. These results were subsequently added to the On-line Encyclopedia of Integer Sequences [5].

In December 2000, Steven J Edwards posted calculations of U(1) to U(6) on the Computer Chess Club bulletin board. However, the value given for U(6), 9417683, did not take into account the possibility that some *en passant* captures may not be possible. If we consider the four move sequences:

A 1. f4 e6 2. Kf2 Qf6 3. f5 g5
B 1. f4 e5 2. Kf2 Qf6 3. f5 g5
C 1. f4 g5 2. Kf2 e6 3. f5 Qf6
D 1. f4 g5 2. Kf2 e5 3. f5 Qf6

we find that there are really only two unique positions as the capture 4. fxg6 (*en passant*) is illegal in positions A and B, and so position A is equivalent to position C, and position B is equivalent to position D. Thus the value given for U(6) in the table above is two less than Edwards' value.

We consider the most popular position at each ply. We note that the position which occurs after 1. e4 e5 2. d4 d5 occurs 14363 times at 8 ply, and the two positions which occur after 1. e4 d6/d5 2. d4 each occur 2600 times at 7 ply. The four positions which occur after 1. e3/e4 e6/e5 each occur 479 times at 6 ply. Similarly the two positions which occur after 1. e3/e4 each occur 87 times at 5 ply. The starting position occurs 16 times at 4 ply.

Further calculations of the number of unique nodes after 8 ply should take into account three-time repetition (and perhaps the 50-move rule), because positions with the same *en passant* status and castling status do not necessarily have the same history. It should be noted that according to the FIDE laws of chess [3], claiming a draw by three-time repetition of the position is voluntary.

Now that the 10 ply calculation is complete, further research could focus on finding the shortest stalemate possible from the beginning of the game. The current record is 19 ply by Sam Loyd.

References

[1] `http://groups.google.com/` Archive of Usenet newsgroups.

[2] `http://www.talkchess.com/` Computer Chess Club, a computer chess discussion board.

[3] `http://www.fide.com/` International Chess Federation webpage.

[4] Robert Hyatt. `ftp://ftp.cis.uab.edu/pub/hyatt/`.

[5] N. J. A. Sloane. `http://www.research.att.com/~njas/sequences/` The On-Line Encyclopedia of Integer Sequences.